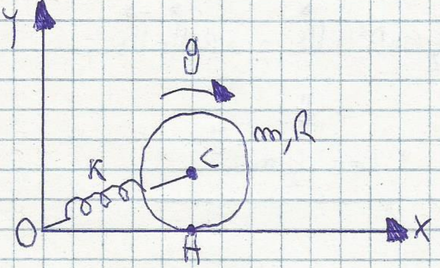
28) Consideriamo il seguente sistema:

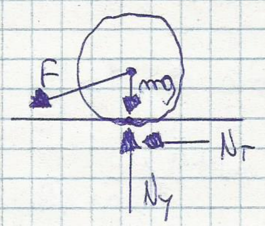


Calcolare le reazioni vincolari, la vel iniziale massima tale per cui si possa avere puro rotolamento. Trovare anche le equazioni del moto.

28) Si scrive:

$V_C = R\dot{\theta}\vec{U}_x$ Il disco rotola e quindi:

$\omega = -\dot{\theta}\vec{U}_z$



La forza elastica vale:

$$F_{el} = k x_C = kR\theta$$

meglio:

$$F_{el} = k(-R\theta\vec{U}_x - R\vec{U}_y)$$

A questo punto ci calcoliamo il momento:

$$K_H = I_H\omega = -\frac{3}{2}mR^2\dot{\theta}\vec{U}_z$$

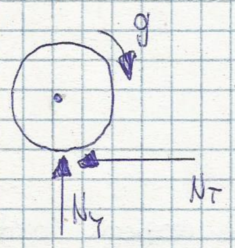
Quindi: $\frac{dK_H}{dt} = \Pi_H + \cancel{V_H}^{\phi} = \Pi_H \Rightarrow -\frac{3}{2}mR^2\ddot{\theta}\vec{U}_z = (C-H)\times F$

$$-\frac{3}{2}mR^2\ddot{\theta}\vec{U}_z = R\vec{U}_y + (-kR\theta\vec{U}_x - kR\vec{U}_y) = kR^2\theta\vec{U}_z$$

Pertanto:

$$+\frac{3}{2}m\ddot{\theta} = k\theta \Rightarrow \frac{3}{2}m\ddot{\theta} + k\theta = \phi$$

Vediamo le reazioni vincolari:



N_T? $\frac{dK_c}{dt} = \cancel{\Gamma_c}^{\,\phi} + R_c \Rightarrow -\frac{1}{2} m R^2 \ddot{\vartheta}\, \vec{U}_z = R_c$

Quindi:

$$-\frac{1}{2} m R^2 \ddot{\vartheta}\, \vec{U}_z = -R \vec{N}_T \vec{U}_z$$

$$\Downarrow$$

$$\underline{N_T = \frac{1}{2} m R \ddot{\vartheta}}$$

Per N_Y: $\frac{dQ_Y}{dt} = \phi \Rightarrow \phi = N_Y - mg - RK$

$$\Downarrow$$

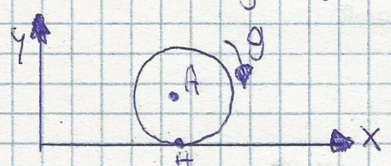
$$\underline{N_Y = mg + RK}$$

Calcoliamoci $V_{0(MAX)}$:

Puro rotolamento $\Rightarrow \mu > \frac{N_T}{N_Y}$ ossia $N_T \le \mu N_Y$. Quindi:

$$\underline{\frac{\frac{1}{2} m R^2 \ddot{\vartheta}}{mg + kR} < \mu}$$

29) Analizziamo il seguente disco in puro rotolamento:



Calcolare $C(t)$ tale per cui: $\vartheta(t) = At^3$

29) Calcoliamo l'energia cinetica:

$T = \frac{1}{2} I_G \omega^2 + \frac{1}{2} m V_G^2$ con $\vec{\omega} = -\dot{\vartheta}\, \vec{U}_z$, $\vec{V}_G = R\dot{\vartheta}\, \vec{U}_x$

$V_G = \phi \Rightarrow T = \frac{1}{2} I_G \omega^2 = \frac{3}{4} m R^2 \dot{\vartheta}^2$

Si ricordi che: $\frac{dT}{dt} = \underbrace{\sum_i \Pi_i}_{\text{Forze attive}} + \sum_i \Pi_i'$ — Potenza forze reattive

con: $\Pi_i = F_i \cdot V_i$

Per i vincoli lisci e bilaterali si ha:

$$N \perp V_i \Rightarrow \Pi_i' = \phi .$$

Quindi:

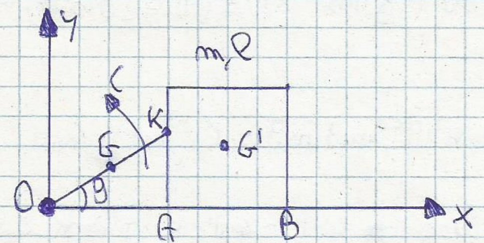
$$\Sigma \Pi = \Pi_g + \Pi_c \quad \text{con:} \quad \Pi_g = -mg\vec{U}_y \cdot V_G = \phi$$
$$\Pi_c = -c(t)\vec{U}_z \cdot (-\dot{\vartheta})\vec{U}_z = c\dot{\vartheta}$$

Pertanto:

$$\frac{dT}{dt} = \frac{3}{2} mR^2 \dot{\vartheta}\ddot{\vartheta} = c\dot{\vartheta}$$
$$\Downarrow$$
$$c(t) = \frac{3}{2} mR^2 \ddot{\vartheta}$$

$$\ddot{\vartheta} = 6At \Rightarrow 2 \quad c(t) = 9mR^2 A$$

3φ) Si consideri:



Calcolare $c(t)$ tale per cui $\dot{\omega} = \phi$ e $N_K(t)$ durante il moto.

3φ) Si ha:

$$\omega = \dot{\vartheta}\vec{U}_z$$
$$V_G = -\frac{\ell}{2}\dot{\vartheta}\,\text{sen}\,\vartheta\,\vec{U}_x + \frac{\ell}{2}\dot{\vartheta}\cos\vartheta\,\vec{U}_y$$
$$V_{G'} = -\ell\dot{\vartheta}\,\text{sen}\,\vartheta\,\vec{U}_x$$

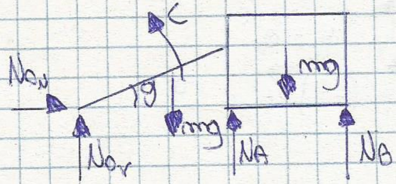
Usiamo il teorema dell'energia cinetica:

$$T = T_{LAMINA} + T_{ASTA} \Rightarrow T_{LAMINA} = \frac{1}{2} m V_{G'}^2$$
$$T_{ASTA} = \frac{1}{2} I_O \omega^2 \Rightarrow T = \frac{1}{2} m V_{G'}^2 + \frac{1}{2} I_O \omega^2$$

Quindi:

$$T = \frac{1}{2} m l^2 \dot{\theta}^2 \operatorname{sen}^2\theta + \frac{1}{6} m l^2 \dot{\theta}^2$$

Scriviamo le potenze:



$$\sum \Pi = \Pi_{gL} + \Pi_{gA} + \Pi_c$$

con:

$$\Pi_{gA} = -mg\vec{u}_y \cdot \left(-\frac{l}{2}\dot{\theta}\operatorname{sen}\theta\,\vec{u}_x + \frac{l}{2}\dot{\theta}\cos\theta\,\vec{u}_y\right)$$

$$\Pi_{gL} = -mg\vec{u}_y \cdot (\dots) = \phi$$

Quindi:

Π_{gA} = potenza forza peso dell'asta $= -mg\frac{l}{2}\dot{\theta}\cos\theta$

Π_{gL} = potenza della forza peso della lamina $= \phi$

Π_c = potenza della coppia $= c\dot{\theta}$

$$\frac{dT}{dt} = m l^2 \dot{\theta}\ddot{\theta}\operatorname{sen}^2\theta + m l^2 \dot{\theta}^2 \operatorname{sen}\theta\cos\theta\,\dot{\theta} + \frac{1}{3} m l^2 \dot{\theta}\ddot{\theta} = \left(-mg\frac{l}{2}\dot{\theta}\cos\theta\right) + c\dot{\theta}$$

Pertanto:

$$\frac{m l^2}{3}\ddot{\theta} + m l^2 \ddot{\theta}\operatorname{sen}^2\theta + m l^2 \dot{\theta}^2 \operatorname{sen}\theta\cos\theta = c - mg\frac{l}{2}\cos\theta$$

La coppia deve essere tale per cui $\dot{\omega} = \phi \Rightarrow \omega(t) = \text{cost}$ e quindi $\ddot{\theta} = \phi$

Pertanto:

$$\dot{\theta} = \omega \quad \text{e} \quad \theta = \omega t \Rightarrow c = m l^2 \omega^2 \operatorname{sen}(\omega t)\cos(\omega t) + mg\frac{l}{2}\cos(\omega t)$$

$$\frac{dQ_x}{dt}_{LAMINA} = F_x^{L} + R_x^{L}$$

LAMINA

$$Q_x = m v_{G'} = -m l \dot{\theta}\operatorname{sen}\theta\,\vec{u}_x$$

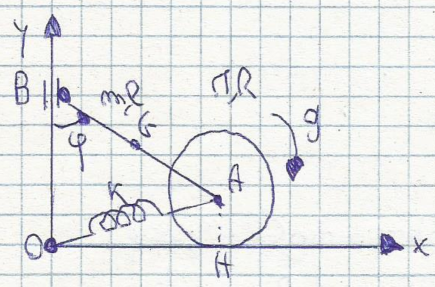
$$R_x = N_K \Rightarrow \left(-m l \dot{\theta}\operatorname{sen}\theta\,\vec{u}_x\right)^{\cdot} = N_K$$

$$F_x = \phi$$

$$\Downarrow$$

$$\left(-m l \ddot{\theta}\operatorname{sen}\theta - m l \dot{\theta}^2\cos\theta\right)\vec{u}_x = N_K\vec{u}_x$$

31) Si consideri:



Calcolare:

1) eq moto

2) $\dot{\theta}, \dot{\varphi}$ quando $\varphi = \frac{\pi}{2}$

31) Per $\varphi(\theta) \Rightarrow \omega_D = -\Omega \vec{U}_z$

$V_A = \ell \dot{\varphi} \cos\varphi \vec{U}_x$

$\dot{\theta} = \frac{\ell}{R} \dot{\varphi} \cos\varphi \quad$ e $\quad V_G = \frac{\ell}{2} \dot{\varphi} \cos\varphi \vec{U}_x - \frac{\ell}{2} \dot{\varphi} \operatorname{sen}\varphi \vec{U}_y$

Scriviamo l'energia cinetica:

$$T = \frac{1}{2} I_H \omega_D^2 + \left(\frac{1}{2} m V_G^2 + \frac{1}{2} I_G \omega_A^2\right)$$

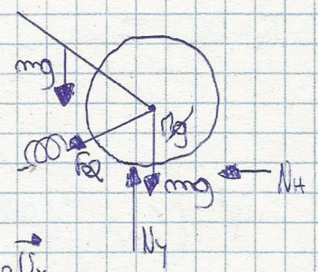
$$= \frac{1}{2}\left(\frac{3}{2} M R^2\right) \dot{\theta}^2 + \left(\frac{1}{2} m \frac{\ell^2}{4} \dot{\varphi}^2 + \frac{1}{2} \frac{1}{12} m \ell^2 \dot{\varphi}^2\right)$$

Quindi:

$$T = \frac{3}{4} M \ell^2 \dot{\varphi} \cos^2\varphi + \left(\frac{1}{8} m \ell^2 \dot{\varphi}^2 + \frac{1}{24} m \ell^2 \dot{\varphi}^2\right) = \frac{3}{4} M \ell^2 \dot{\varphi}^2 \cos^2\varphi + \frac{1}{6} m \ell^2 \dot{\varphi}^2$$

Ora calcoliamo:

$$\Pi_{ASTA} = -mg \vec{U}_y \vec{V}_G = mg \frac{\ell}{2} \dot{\varphi} \operatorname{sen}\varphi$$



$$\Pi_{ELASTICA} = k(O-A) \vec{V}_A =$$

$$= k\left(-\ell \operatorname{sen}\varphi \vec{U}_x - R \vec{U}_y\right) V_A$$

$$= k\left(-\ell \operatorname{sen}\varphi \vec{U}_x - R \vec{U}_y\right) \ell \dot{\varphi} \cos\varphi \vec{U}_x$$

$$= k \ell^2 \dot{\varphi} \operatorname{sen}\varphi \cos\varphi$$

Deriviamo:

$$\frac{dT}{dt} = \frac{3}{2} M \ell^2 \dot{\varphi} \ddot{\varphi} \cos^2\varphi - \frac{3}{2} M \ell^2 \dot{\varphi}^3 \cos\varphi \operatorname{sen}\varphi + \frac{1}{3} m \ell^2 \dot{\varphi} \ddot{\varphi} =$$

$$= \frac{1}{2} mg\ell \dot{\varphi} \operatorname{sen}\varphi - k\ell^2 \dot{\varphi} \operatorname{sen}\varphi \cos\varphi$$

Vediamo il secondo punto del problema:

$$E_m = E_c + E_p = cost.$$

Meglio:

$$T + U = cost \Rightarrow T + U = T_0 + U_0$$

Quindi:

$$U = U_{ASTA} + \overset{\text{non serve}}{\cancel{U_{DISCO}}} + U_{EL.}$$

con:

$$U_{ASTA} = mg\left(R + \frac{\ell}{2}\cos\varphi\right)$$

$$U_{DISCO} = MgR \to cost.$$

$$U_{ELASTICA} = \frac{1}{2}k\left(\ell^2 \sin^2\varphi + R^2\right)$$

Pertanto:

$$U = mg\left(\frac{\ell}{2}\cos\varphi\right) + \frac{1}{2}k\ell^2\sin^2\varphi + cost.$$

Ora:

$$T + U = \frac{3}{4}M\ell^2\dot\varphi^2\cos^2\varphi + \frac{1}{6}m\ell^2\dot\varphi^2 + mg\frac{\ell}{2}\cos\varphi + \frac{1}{2}k\ell^2\sin^2\varphi$$

Quando:

$$\varphi = 0 \Rightarrow T_0 + U_0 = \frac{3}{4}M\ell^2\left(\frac{R\Omega}{\ell}\right)^2 + \frac{1}{6}m\ell^2\left(\frac{R\Omega}{\ell}\right)^2 + mg\frac{\ell}{2}$$

$$\dot\vartheta_0 = \Omega \Rightarrow \dot\varphi(0) = R\Omega/\ell \qquad \text{NB: } \dot\varphi = \frac{R\dot\vartheta}{\ell\cos\varphi}$$

Pertanto:

$$T_0 + U_0 = T + U\big|_{\varphi = \frac{\pi}{2}} \Rightarrow \frac{3}{4}M\ell^2\left(\frac{R\Omega}{\ell}\right)^2 + \frac{1}{6}m\ell^2\left(\frac{R\Omega}{\ell}\right)^2 + mg\frac{\ell}{2} =$$

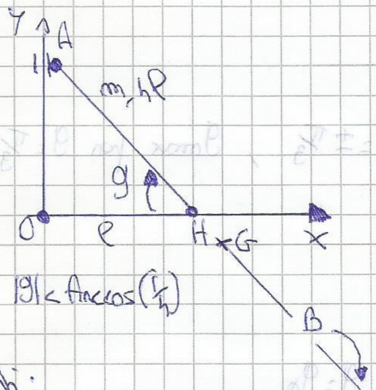
$$= \frac{1}{6}m\ell^2\dot\varphi^2 + \frac{1}{2}k\ell^2$$

Conclusione:

$$\dot\varphi^{\cancel{2}} = \sqrt{\frac{6}{m\ell^2}\left(\frac{3}{4}MR^2\Omega^2 + \frac{1}{6}mR^2\Omega^2 + mg\frac{\ell}{2} - \frac{1}{2}k\ell^2\right)}$$

$$\dot\vartheta = \frac{\ell}{R}\dot\varphi\cos\varphi = \frac{\ell}{R}\sqrt{\cdots}\,\cos\varphi = 0$$

32) Si consideri:



$|\theta| < \arccos\left(\frac{1}{h}\right)$

Calcolare: 1) $\dot{\theta}(\theta)$
2) $R_{Ox}(\theta)$

condizioni iniziali:
$\theta(0) = 0$
$\dot{\theta}(0) = 0$

32) Quindi:

$$U+T \Rightarrow \begin{cases} x_G = 2\ell\cos\theta \\ y_G = y_A - 2\ell\operatorname{sen}\theta = \ell\operatorname{Tg}\theta - 2\ell\operatorname{sen}\theta \end{cases}$$

Calcoliamo la velocità V_G:

$$\dot{G} = \begin{cases} \dot{x}_G = -2\ell\dot{\theta}\operatorname{sen}\theta \\ \dot{y}_G = \ell/\cos\theta\,\dot{\theta} - 2\ell\dot{\theta}\cos\theta \end{cases} \Rightarrow T = \frac{1}{2}mV_G^2 + \frac{1}{2}I_G\dot{\theta}^2$$

$$\Downarrow$$

$$T = \frac{1}{2}m\left(h\ell^2\dot{\theta}^2\operatorname{sen}^2\theta + \frac{\ell^2}{\cos^2\theta}\dot{\theta}^2 + h\ell^2\dot{\theta}^2\cos^2\theta - \frac{h\ell^2}{\cos\theta}\dot{\theta}^2\right) + \frac{1}{2}\frac{1}{12}m(h\ell)^2\dot{\theta}^2$$

$$= \frac{1}{2}m\ell^2\dot{\theta}^2\left(h + \frac{1}{\cos^2\theta} - \frac{h}{\cos\theta}\right) + \frac{1}{3}m\ell^2\dot{\theta}^2 = m\ell^2\dot{\theta}^2\left(\frac{8}{3} + \frac{1}{2\cos^2\theta} - \frac{2}{\cos\theta}\right)$$

Quindi:

$$V = mgy_G = mg(\ell\operatorname{Tg}\theta - 2\ell\operatorname{sen}\theta)$$

$$\Downarrow$$

$$E_m = U+T = mg(\ell\operatorname{Tg}\theta - 2\ell\operatorname{sen}\theta) + m\ell^2\dot{\theta}^2\left(\frac{8}{3} + \frac{1}{2\cos^2\theta} - \frac{2}{\cos\theta}\right) = \text{cost}$$

Si ha:

$$E_{m,0} = U_0 + T_0 = 0 \Rightarrow \dot{\theta}^2 = -mg\ell(\operatorname{Tg}\theta - 2\operatorname{sen}\theta) \Big/ m\ell^2\left(\frac{8}{3} + \frac{1}{2\cos^2\theta} - \frac{2}{\cos\theta}\right)$$

$$\Downarrow$$

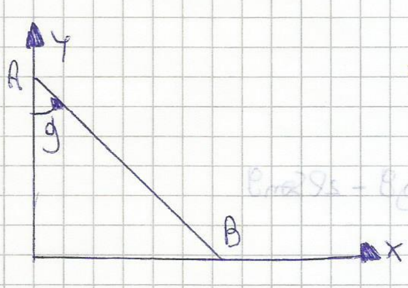
$$\dot{\theta} = \sqrt{\ldots}$$

Per il secondo punto θ è massimo quando $\dot\theta = 0$. Quindi:

$$\operatorname{sen}\theta = 0 \quad \begin{cases} \theta = 0 \\ \theta = \pi \text{ (non possibile)} \end{cases}$$

$$2 - 1/\cos\theta = 0 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \pm\frac{\pi}{3}, \quad \theta_{max} \text{ per } \theta = \pi/3.$$

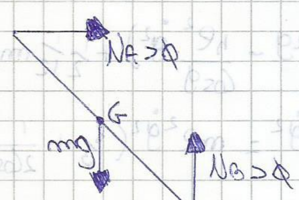
33) Consideriamo:



- All'inizio $\theta = \theta_0$
  $\dot\theta(0) = 0$

Calcolare θ, tale per cui ho il distacco

33) Disegniamo il diagramma del corpo rigido:



$N_A = 0$ e $m\ddot{x}_G = N_A$ → prima eq. cardinale

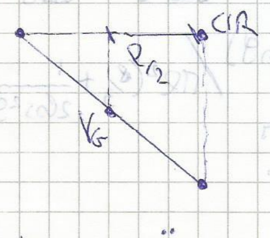
→ condizione di distacco

$x_G = \frac{l}{2}\operatorname{sen}\theta$

$\dot{x}_G = \frac{l}{2}\dot\theta\cos\theta$

$$\ddot{x}_G = \frac{l}{2}\ddot\theta\cos\theta - \frac{l}{2}\dot\theta^2\operatorname{sen}\theta \Rightarrow \ddot\theta\cos\theta = \dot\theta^2\operatorname{sen}\theta$$

Dobbiamo calcolare $\ddot\theta$:



$\vec{V}_{CIR} = 2\vec{V}_G \qquad V_{CIR} /\!/ V_G \Rightarrow I_{CIR}\ddot\theta = \frac{l}{2} mg\operatorname{sen}\theta$

$\Downarrow$

$$I_{CIR} = I_G + m d_{CG}^2 = \frac{1}{12}ml^2 + \frac{1}{4}ml^2 = \frac{1}{3}ml^2$$

Quindi: $\frac{1}{3}ml^2\ddot\theta = \frac{l}{2} mg\operatorname{sen}\theta \Rightarrow \ddot\theta = \dots$